Thai Journal of Development Administration Vol. 27, No. 1, January 1987

A NOTE ON THE ESTIMATION OF BENCHMARK CAPITAL STOCK*

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Often when working with a macro-econometric model, a series of aggregate capital stock needs to be constructed.[1] The best known technique for measuring capital stock is *the Perpetual Inventory Method,* which typically assumes that the stock is depreciated at a constant rate, say "d" per annum, and thus has a theoretically infinite life. The determination of capital stock and its depreciation can be expressed as follows :

$$K_t = K_{t-1} + I_t - D_t \qquad \ldots (1)$$

$$D_t = d.K_{t-1} \qquad \ldots (2)$$

where K_t = capital stock measured at the end of period at constant prices (deflated by the fixed investment deflator)

I_t = gross fixed investment at constant prices

D_t = depreciation at constant prices (deflated by the fixed investment deflator).

If an initial (or benchmark) value for the stock can be obtained (possibly from other sources) and an appropriate value of "d" can be assumed, then the time series of capital stock can be estimated from equations (1) and (2). [2]/[3]/ But if a benchmark cannot be obtained, or if the benchmark is doubtful, then the following is an alternative method which may be applicable. Lagging equation (2) by one period :[4]

$$D_{t-1} = d.K_{t-2} \qquad \ldots (3)$$

Subtracting equation (3) from equation (2), to obtain

$$D_t - D_{t-1} = d.(K_{t-1} - K_{t-2}) \qquad \ldots (4)$$

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*This note was adopted from "Appendix 3" of my Ph.D. dissertation, **A Macro-Econometric Model of Thailand,** McMaster University, June 1983. The author hoped that it may provide additional knowledge on capital stock estimation.

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Then, lagging equation (1) by one period and re-arranging it :

$$K_{t-1} - K_{t-2} = I_{t-1} - D_{t-1} \quad \ldots (5)$$

Substituting (5) into (4), to obtain

$$D_t - D_{t-1} = d.(I_{t-1} - D_{t-1}) \quad \ldots (6a)$$

or

$$D_t = d.(I_{t-1} - D_{t-1}) + D_{t-1} \quad \ldots (6b)$$

Without any difficulty, a Single-equation regression method can be applied to obtain $\hat{d}$, an estimate of d from either equation (6a) or (6b), then $\hat{d}$ can be used to obtain a benchmark capital stock by inverting equation (2) and rewriting it as

$$K_{t-1} = D_{t/\hat{d}} \quad \ldots (7)$$

Two options are opened for estimating the stock series : equation (1) can be used by applying actual depreciation with the benchmark from (7); or equation (8) is applied, again using the benchmark from (7) :

$$K_t = (1 - \hat{d}) .k_{t-1} + I_t \quad \ldots (8)$$

Two points should be noted. First, the benchmark stock should be derived from different years so as to permit experimentation with alternative initial capital stocks. It has been found (Choudhry, 1972, p. 151) that "...forward calculation from an early benchmark was better than the other way around, because the error in the estimate of the benchmarks tends to expand as we move back, and contract as we move forward. So that as one moves forward, the series becomes less sensitive to a mismeasurement of the benchmark...".

Second, the data used for depreciation are capital consumption allowance estimates from national accounts, and these data may be of doubtful accuracy; in as much as they do not represent real depreciation. The standard method of estimating capital consumption allowances, as described in the U.N. Yearbook of National Accounts Statistics, is as follows : "Consumption of fixed capital is calculated from the charges for wear and tear, foreseen obsolescence and the expected rate of accidental damage not made good by repair in all fixed assets, valued at current replacement cost. Charges are not included in respect of unforeseen obsolescence or the depletion of natural resources." Although the method for calculating depreciation may be considered satisfactory, measurement error may, of course, be present for various reasons in actual data.

Empirical results : Thailand's Data

Equation (6b) was applied to estimate the values of "d" in the agricultural and non-agricultural sectors of Thailand. The equations fitted to 1954–78 annual data are as follows :

$$Da_t = .037359\ (Ia_{t-1} - Da_{t-1}) + Da_{t-1} \quad \text{... (9)}$$
$$(2.3875)$$

$\bar{R}^2$ = .9125; Se = 140.06; D.W. = 1.9645;

$$Dm_t = .040182\ (Im_{t-1} - DM_{t-1}) + Dm_{t-1} \quad \text{... (10)}$$
$$(5.1788)$$

$\bar{R}^2$ = .9793; Se = 470.59; D.W. = 2.2233

Where "a" and "m" represent the agricultural and non-agricultural sectors, respectively. D.W. is the Durbin-Watson statistic. Se is the standard error of estimate. The figures in brackets are t-ratios. The Cochrane-Orcutt method was applied in order to correct for first-order autocorrelation, which was prominent in the OLS results.

The estimated rates of depreciation are 0.037359 in the agricultural sector and 0.040182 in the non-agricultural sector. These estimates seem quite reasonable.

Equation (7) was then applied to obtain benchmark estimates of capital stock for 1952 :

$$\begin{aligned} Ka_{1952} &= (1/0.037359)\ Da_{1953} \\ &= (1/0.037359)\ (326.95) \\ &= 8{,}751.57 \text{ millions of 1972 baht.} \\ Km_{1952} &= (1/0.040182)\ Dm_{1953} \\ &= 27{,}880.64 \text{ millions of 1972 baht.} \end{aligned}$$

The series of stock estimates based on equation (1), and using these benchmarks, are reported for comparison with the estimates by Trescott (1967) and Chaipravat et at. (1979) in Tables 1 and 2. Note that Trescott used a survey method with assumptions about the rates of depreciation and some other assumptions. His series were for the period 1945-65, in 1956 prices, the series were converted into 1972 prices. His series of both stocks were accumulated from benchmarks for 1952-53.

From Tables 1 and 2, it was found that our 1953 estimates of both capital stocks are very close to those of Trescott. However, his stocks grow more slowly. This may be due to his assumptions about the rates of depreciation, which are higher than our estimated rates.

The series of capital stocks developed by Chaipravat et al. may be too high, since their total capital-output ratio was sometimes greater than 6. The usual range of the total capital-output ratio is between 2.5 and 3.5. (See Taylor, 1979, pp.87-88.)

Footnotes

[1]The series of aggregate capital stock may, in fact, not exist for a number of reasons as indicated during a series of debates between the two Cambridges on capital theory (see Harcourt, 1972; Jones, 1979). But to consider something concrete about the marginal productivity of capital, capital-output ratio, and so on, an aggregate series of capital stock seems to be the only tool available.

[2]The usual method for estimating a capital stock benchmark can be found in Brown (1964) and Rymes (1967). It involves the appropriate assumption of depreciation rates. For example, we may assume that machinery and equipment have an average economic life of 20 years. With linear depreciation, a piece of capital depreciates one twentieth of its value a year, and hence at the end of its twentieth year has no value left. We thus have the following formula for the calculation of benchmark capital :

$$K_t = I_t + (N-1)/N.I_{t-1} + (N-2)/N.I_{t-2} + \cdots$$
$$\cdots + 2/N.I_{t-N+2} + 1/N.I_{t-N+1}$$

where N = average life of capital.

[3]Harberger (1974) suggested that benchmark capital may be obtained by the following formula :

$$K = I/(g_y + d)$$

where g_y is the rate of growth of output and d is to be appropriately assumed. This method is suitable provided that the economy grew at a fairly constant rate over the period of estimation.

[4]Depreciation might be assumed as $D_t = d.(K_t + K_{t-1})/2$ in the model. The estimated result would then be slightly different.

Table 1 Capital Stocks and Capital-Output Ratios in Agriculture (in billions of baht, at 1972 prices)

Year	Ka	Ka!	Ka +	Ka/Ya	Ka !/Ya	Ka +/Ya	Ya
1952	8.7516	9.7469	–	.4637	.5165	–	18.872
1953	9.6191	10.153	–	.4419	.4664	–	21.768
1954	10.491	10.559	–	.5202	.5236	–	20.168
1955	11.341	10.830	–	.4935	.4713	–	22.981
1956	12.206	11.236	–	.5216	.4802	–	23.400
1957	13.255	11.777	–	.5772	.5128	–	22.965
1958	14.527	12.454	–	.6029	.5169	–	24.096
1959	16.096	13.267	–	.6492	.5351	–	24.794
1960	17.966	14.485	46.332	.6365	.5132	1.6414	28.227
1961	19.809	15.703	45.636	.6799	.5390	1.5664	29.135
1962	21.788	17.057	45.201	.6954	.5444	1.4427	31.330
1963	23.945	18.817	45.147	.7020	.5517	1.3236	34.110
1964	26.116	20.441	45.408	.7546	.5906	1.3120	34.610
1965	28.530	21.930	45.764	.7940	.6104	1.2737	35.931
1966	30.928	–	46.523	.7567	–	1.1382	40.873
1967	33.299	–	48.231	.8359	–	1.2108	39.834
1968	35.604	–	50.067	.8146	–	1.1455	43.706
1969	37.663	–	51.658	.8010	–	1.0987	47.018
1970	40.090	–	53.816	.8295	–	1.1135	48.332
1971	42.120	–	55.800	.8335	–	1.1041	50.537
1972	44.540	–	56.951	.8923	–	1.1409	49.919
1973	46.887	–	58.474	.8337	–	1.0398	56.237
1974	49.605	–	60.582	.8708	–	1.0636	56.962
1975	52.503	–	63.175	.8457	–	1.0176	62.081
1976	55.743	–	66.421	.8459	–	1.0079	65.898
1977	60.484	–	70.323	.9229	–	1.0730	65.537
1978	65.676	–	74.556	.8750	–	.9933	75.059

where Ka = our estimate of fixed capital stock in agriculture

Ka ! = Trescott estimate of fixed capital stock in agriculture

Ka + = Chaipravat et al. estimate of fixed capital stock in agriculture

Ya = agricultural value added output.

Table 2 Capital Stocks and Capital-Output Ratios in Non-Agriculture (in billions of baht, at 1972 prices)

Year	Km	Km!	Km +	Km/Ym	Km!/Ym	Km +/Ym	Ym
1952	27.881	29.241	–	1.1681	1.2251	–	23.868
1953	32.131	32.219	–	1.2306	1.2340	–	26.110
1954	36.520	34.926	–	1.3462	1.2875	–	27.128
1955	40.380	37.769	–	1.3752	1.2863	–	29.362
1956	44.456	40.747	–	1.4605	1.3387	–	30.438
1957	50.585	45.350	–	1.6112	1.4444	–	31.396
1958	56.218	49.547	–	1.7616	1.5525	–	31.913
1959	62.517	54.149	–	1.6861	1.4605	–	37.077
1960	70.021	59.835	419.47	1.6707	1.4276	10.008	41.912
1961	77.627	66.468	414.33	1.7358	1.4863	9.2647	44.721
1962	87.030	74.455	411.38	1.7941	1.5349	8.4806	48.508
1963	99.012	85.420	410.88	1.8883	1.6291	7.8361	52.434
1964	112.58	97.604	412.79	1.9530	1.6932	7.1608	57.646
1965	126.47	110.33	415.51	1.9880	1.7344	6.5319	63.613
1966	144.44	–	421.59	2.0397	–	5.9534	70.815
1967	166.35	–	431.86	2.0650	–	5.3611	80.555
1968	189.73	–	444.89	2.1835	–	5.1200	86.892
1969	216.02	–	461.08	2.2999	–	4.9091	93.923
1970	241.10	–	478.12	2.3693	–	4.6985	101.76
1971	264.13	–	492.46	2.4789	–	4.6218	106.55
1972	283.72	–	506.72	2.4735	–	4.4175	114.71
1973	306.33	–	524.49	2.4722	–	4.2329	123.91
1974	333.07	–	544.07	2.5045	–	4.0911	132.99
1975	359.10	–	565.12	2.5390	–	3.9956	141.43
1976	386.18	–	587.55	2.4658	–	3.7517	156.61
1977	423.91	–	619.07	2.4461	–	3.5721	173.30
1978	464.47	–	654.77	2.4219	–	3.4142	191.78

where Km = our estimate of fixed capital stock in non-agriculture

Km! = Trescott estimate of fixed capital stock in non-agriculture

Km + = Chaipravat et al. estimate of fixed capital stock in non-agriculture

Ym = non-agricultural value added output.

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