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IMPACT OF UNCERTAINTY OF INPUT SUPPLY ON LOCATIONS OF MANUFACTURING PLANTS: A THEORETICAL ANALYSIS

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I. Introduction

The uncertainty of the input supply has been recognised as causing manufacturers to relocate from locations of unreliable input supply to more reliable ones. For example, Tolley, Upton and Hastings (1977) found that electricity outages caused unemployment in the industries whose electricity was curtailed, as well as in other industries via multiplier effects. They sometimes caused the migration of unemployed workers, as well as the relocation of manufacturing plants to sites with a more reliable supply of electricity. The survey study of Schmenner (1982) found that unionization is one of the most important factors influencing choice of location of manufacturing plants. This was confirmed by Wheat (1985) who indicated that unionization, among other things, was one of the important factors that caused the substantial manufacturing growth in the southern and western regions of the United States during the period of 1963 to 1977.

The uncertainty of the input supply is also a practical issue, as it can be seen that several regional development plans have been conducted on the basis of a cheap and reliable energy supply: the development of the Tennessee Valley Authority, the Central Valley Project in California and the Columbia River Development in the Pacific Northwest, for example.

Although the effects of uncertainty of input supply on production, employment and location of manufacturers have been realized empirically and by policy makers, such effects have not yet been explained theoretically. Most location theories aim to explain the impact of the uncertainty of other factors such as input price and demand. Most of these models are based on the location-production literature.

The main purpose of this study is to theoretically investigate the influence of the uncertainty or stochasticity of the input supply on location of manufacturing plants, as suggested by various empirical studies and regional development policies. Similarly to many stochastic location models, the model of this study is also based on location-production literature.

The location-production literature can be dated back to the theoretical contributions of Weber (1929) and Moses (1958), among others. According to Weber, the optimal location of a single plant firm in heterogeneous space is related to market and raw materials in economic space. Output and raw materials must be transported from plant to market and from raw material sources to the plant, respectively. The optimal location can be determined by minimizing the transportation cost of all inputs and outputs. Following the location theory of Weber, Moses (1958) explicitly incorporated the neoclassical production theory into Weber's triangular model. In his analysis, the distance between plant and market site was assumed to be given. The optimal location, optimal output level and optimal input ratios were determined simultaneously at the point where the marginal rate of technical substitution was equal to the (delivered) input price ratio. It is the contribution of Moses (1958) that gives the name "location-production" to this literature.

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Following the work of Moses, many location-production models were developed, mainly to examine the conditions of separability between location and production under various assumptions. These assumptions include variable distance between plant site and market site in a triangular model (Sakashita (1968), Bradfield (1971), Khalili, Mathur and Bodenhorn (1974)); n inputs at n different sites with given demand (Hurter and Wendell (1972) and Hurter, Martinich and Venta (1980)); variable transportation rate of output with given demand (Woodward (1973)); normal demand function with two inputs and one output (Thisse and Perreur (1977)); variable transportation rates of output and inputs with normal demand (Miller and Jensen (1978) and Mathur (1979)); and the reaction of rival firms (Kusumoto (1984)).

There are abundant location-production models related to uncertainty of input price and demand [e.g. Mai (1975), Cooper (1974, 1978), Hsu and Mai (1975), Mathur (1983), Martinich (1980) and Martinich and Hurter (1982, 1985a, 1985b)]. Yet the uncertainty or stochasticity of input supply has not been incorporated into the location-production model, which this study intends to do.

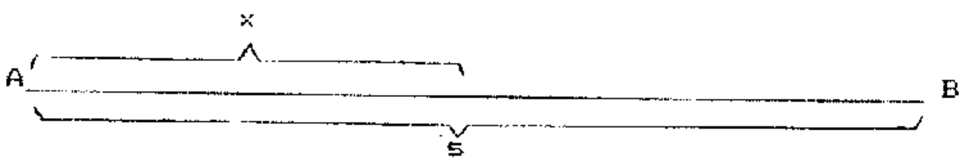
II. The Model

The model is based on the location-production theory with one stochastic input. The theoretical model considers the case of a single plant firm with no relocation cost and perfect and costless information. The typical firm is assumed to use two inputs, Z_1 and Z_2, in the production of a single output Z_0, with neoclassical production function. The production function is at least twice differentiable and quasi-concave. Z_1 is stochastic while Z_2 is deterministic. The production function can be given as:

(1) $Z_0^{\#} = F[Z_1^{\#}, Z_2]$,

where $^{\#}$ denotes the stochasticity of variables.

Z_1 and Z_2 are located at two different locations, A and B, respectively. The market is assumed to be at point A, source of Z_1. The firm chooses a location along the linear market AB with distance x from A. Z_1 and output are transported from input site to plant and from plant to market at point A. Z_2 is transported from B to plant at distance s-x.



The supply of stochastic input $Z_1^{\#}$ is shown as:

(2) $Z_1^{\#} = Z_1 - \tau g K^{\#}$,

where

(3) $g = g(x)$,

(4) $\tau = \tau(Z_1)$,

(5) $K^{\#} = K + e$,

where Z_1 is the quantity of stochastic input used in the production in case of no curtailment of its supply. τ is the average quantity of Z_1 cut down in each supply shortfall and assumed to be a positive function of Z_1. The stochastic component $K^{\#}$ consists of two terms: K and e; K is the expected value of $K^{\#}$ whereas e is the purely random term. It is assumed that expected value of e is equal to zero.

(6) $Ee = 0$.

The product of these three terms, τ, g, $K^{\#}$, is the total quantity of Z_1 curtailed in each period.

The firm in this model is assumed to maximize its expected utility, $E[U(\pi^{\#})]$, where utility is specified as a function of the stochastic profit. This utility function, $U(\pi^{\#})$, satisfies all the von Neumann-Morgenstern axioms, i.e. $U' > 0$ and $U'' < 0$. (We consider only the risk averse firm.)

The expected utility can be presented as:

$$(7)\quad E(U(\pi^{\#})) = E\{U[(P_0 - C_0)F^{\#}(Z_1{}^{\#}, Z_2) - ((P_1 + C_1)Z_1{}^{\#} - (P_2 + C_2)Z_2]\}$$
$$= E\{U[P_0F^{\#}(Z_1 - \tau(Z_1)g(x)K^{\#}, Z_2) - P_1(Z_1 - \tau(Z_1)g(x)K^{\#}) - P_2Z_2\},$$

where $\pi^{\#}$ is the stochastic profit; P_0 is the price of output which is assumed to be given at market point (A); P_1 and P_2 are mill prices of Z_1 and Z_2, respectively, which are given at their sources; C_0, C_1 and C_2 are transportation costs of Z_0, Z_1 and Z_2, respectively, and $C_0'(x) > 0$, $C_1'(x) > 0$ and $C_2'(x) < 0$. Delivered prices of inputs or prices of inputs paid by firm are equal to their mill prices plus transportation costs. It is assumed that there is a unique interior solution for the optimal location.

The optimal solution can be found by maximizing expected utility in equation 7 subject to the three choice variables: Z_1, Z_2 and x. Therefore, there are three first-order conditions as follows:

$$(8)\quad \frac{\delta[EU(\pi^{\#})]}{\delta(Z_1)} = EU'(P_0F_1{}^{\#} - P_1)(1 - gK^{\#}\tau') = 0$$

$$(9)\quad \frac{\delta[EU(\pi^{\#})]}{\delta(Z_2)} = EU'(P_0F_2 - P_2) = 0$$

$$(10)\quad \frac{\delta[EU(\pi^{\#})]}{\delta(x)} = EU'[-C_0'Z_0{}^{\#} - (P_0F1^{\#} - P_1)\tau K^{\#}g' - C_1'Z_1 + C_1'\tau K^{\#}g - C_2'Z_2\},$$
$$= 0$$

F1 and F2 are marginal productivities of Z_1 and Z_2, respectively. It should be noted that $F1^{\#}$ is stochastic because of the stochasticity of Z_1. Assuming that the productivity of Z_1 depends on the stochasticity of Z_1 but the productivity of Z_2 does not. Therefore, the second derivative of Z_2 with respect to Z_2 (F22) and its cross productivity (F12 or F21) do not depend on the stochasticity of Z_1. The reason is that the stochasticity of Z_1 has only small or no effect on the cross productivity and productivity of Z_2.

The second-order conditions for the maximization of $E[U[\pi^{\#}]]$ are given by:

$$(11) \quad \delta^2[E(U)]/\delta(Z_1)^2 = \{E[U'[P_0F11^{\#}(1 - gK^{\#}\tau')^2 + E[U''[G]]^2\} < 0$$

$$(12) \quad \delta^2[E(U)]/\delta(Z_2)^2 = \{E[U'[P_0F22]]\} < 0$$

$$(13) \quad \delta^2[E(U)]/\delta(x)^2 = \{E[U'[2C_0'F1^{\#}K^{\#}\tau g' + 2C_1'\tau K^{\#}g' + P_0F11^{\#}(\tau K^{\#}g')^2]] + EU''[D]^2\} < 0$$

$$(14) \quad \{E[U'[P_0F11^{\#}(1-gK^{\#}\tau')^2] + E[U''[G]^2]\}\{E[U'[P_0F22]]\} - \{E[U'[P_0F12(1-gK^{\#}\tau')]]\}^2 > 0$$

and

$$(15) \quad \{N\} < 0.$$

{N} needs explanation. First, taking the total differential of the first order conditions, the determinant of the coefficients of all endogenous variables is {N}.

From the first-order conditions, the function of the optimal location can be given by:

$$(16) \quad x^E - x^E(P_0,P_1,P_2,C_0,C_1,C_2,C_0',C_1',C_2',K,\tau,g,\tau',g']$$

where ^{E} denotes the optimal value of the variable. Since location of manufacturing plants depends mostly on relative prices of output and inputs, comparing alternative locations, not the actual values of variables, the optimal location may be specified as:

$$(17) \quad x^E - x^E[C_0',C_1'C_2',K,\tau,g,\tau',g']$$

III. Comparative Static Analysis

Theorem 1. An increase in the uncertainty of the supply of input at a location, relative to the uncertainty at or near the source of the input, induces a firm at a location distant from the source of the input to relocate to the source or near the source of the input.

This theorem emphasizes the impact of the change in the **relative uncertainty** of the supply of an input (Z_1) at locations distant from the source of the input, relative to locations near the source of the input. The analysis is aimed at firms located at locations further away from the source of an input with uncertain supply (e.g. natural gas).

The comparative static analysis investigates the change in the location of the firm, represented by the distance of the firm from the source of Z_1, when the relative reliability of Z_1 changes in such a way that the increase in the uncertainty of the supply of Z_1 is greater the more distant the location is from the source of the input Z_1, relative to the nearby location. Specifically, the analysis investigates the change in x as g' increases.

The comparative static analysis of the change in the location of the firm (x) with respect to the change in g′ is represented by:

$$(18)\quad \frac{\delta(x^E)}{\delta(g')} = \{ [\{[EU'[P_0F11^{\#}(1 - gK^{\#}\tau')^2 + EU''[G]^2][EU'[P_0F22]]\} - [\{EU'[P_0F12(1 - gK^{\#}\tau')]\}^2][EU'[P_0F1^{\#} - P_1]\ \tau K^{\#}\} \}/\ \{N\}$$

where {N} is as specified in equation 15. The numerator is the product of two terms. The first term of the numerator is positive, from the second-order condition 14. The second term of the numerator is also positive. Therefore, the numerator is positive. From the second-order condition (equation 11), {N} is negative. The sign of $[\delta(x)/\delta(g')]$ is, therefore, negative.

The analysis suggests that an increase in the relative expected number and quantity of the curtailment and uncertainty of the supply of stochastic input (Z_1) at the distant location, relative to the location near to the source of Z_1, induces the firm to relocate to a location near the source of Z_1. Most likely, Z_1 or natural gas intensive firms will be affected most by an increase in the relative uncertainty of the supply of natural gas. The support for this argument in the model is that the second term of the numerator, $EU'\ [(P_0F1^{\#} - P_1)K\tau]$, increases with the factor τ. This term represents the average quantity of Z_1 or natural gas curtailed in each cut down of the input supply which increases with the quantity of usage of the input. For otherwise identical firms, the firm using more Z_1 (or natural gas-intensive firm) would use more natural gas. As a result, that firm will have the higher τ and is more affected by the curtailment of Z_1.

Theorem 2. An increase in the transportation rate (or interlocational price differential of the input with uncertain supply) at the distant location relative to the source of the input with uncertain supply, Z_1, induces the firm to relocate to the source of the input with uncertain supply, Z_1.

The proof of this theorem can be obtained from the derivative of x with respect to C_1'. The derivative is given by:

$$(19)\quad \frac{\delta(x^E)}{\delta(C_1')} = \{[EU'Z_1^{\#}][EU'P_0F11^{\#}(1 - gK^{\#}\tau')^2 + EU''G^2][EU'P_0F22] - [EU'P_0F12(1 - gK^{\#}\tau')]^2\}/\{N\}.$$

From the second-order condition, {N} is negative. The numerator is the product of two terms. The first term, $EU'Z_1^{\#}$ is positive. The second term is positive from the first-order condition. The numerator is, therefore, positive and the derivative is then negative. This implies that the increase in C_1' (the interlocational price differential or transportation rate of the input with uncertain supply) induces the firm to relocate to the source of the input.

Theorem 3. An increase in the transportation rate or interlocational price differential of other inputs, with deterministic supply, which are located at sites other than that of the source of the input with uncertain supply, induces the firm to relocate to the source of the deterministic input supply and away from the source of the input with uncertain supply.

To prove this theorem, it must be shown that the derivative of x with respect to C_2' is negative. It implies that the lower value of C_2' (greater absolute value of C_2') produces the greater value of x (distance of firm to the source of the input with uncertain supply).

The derivative is given by:

$$(20)\quad \frac{\delta(x^E)}{\delta(C_2')} = \{[EU'Z_2][[EU'P_0F11^{\#}(1 - gK^{\#}\tau')^2 + EU''G^2][\{EU'P_0F22] - [EU'P_0F12(1 - gK^{\#}\tau')]^2\}/\{N\}.$$

Similarly to Theorem 2, the denominator is negative and the numerator is positive. The numerator is composed of the product of two terms. The first term, $EU'Z_2$, is positive. The second term is also positive from the second order condition. This theorem is, therefore, proven.

Theorem 4. An increase in demand for output at a location relative to other locations induces the firm to relocate to the location with high demand growth.

An increase in demand for output, say, at the source of the input with uncertain supply may be represented by an increase in price of the output at that location relative to other locations. In this model, an increase in relative demand at the source of the input with uncertain supply may be represented by the increase in C_0'. The reason is that, in this model, the price per unit received by the firm is the mill price, $P_0 - C_0$. An increase in C_0' implies that the price received by a firm at the source of the input, Z_1, increases relative to other locations. This situation is analogously the same as an increase in demand relative to other locations. The theorem is true when the derivative of x with respect to C_0' is negative. The derivative is given by:

$$(21)\quad \frac{\delta(x^E)}{\delta(C_0')} = \{[EU'Z_0^{\#}][[EU'P_0F11^{\#}(1 - gK^{\#}\tau')^2 + EU'G^2][EU'P_0F22] - [EU'P_0F12(1 - gK^{\#}\tau')]^2\}/\{N\}.$$

The derivative is negative because the denominator and numerator are positive and negative, respectively. The analysis suggests that the growth of demand for manufacturing output at a particular location, in this case at the site of the input with uncertain supply, could induce the firm to relocate to that location.

IV Summary

The study investigates theoretically the impact of the uncertainly of input supply on location of a manufacturing plant, on the basis of the location-production literature. The model is based on two assumptions which are different from most location-production models, namely, stochastic supply of an input and maximization of the expected utility of a firm. The theoretical model supports some of the empirical studies, that the uncertainty of the input supply has an impact on location of a manufacturing plant. It causes a manufacturing plant to relocate toward a location with a more certain input supply. Moreover, the predictions from the model are consistent with those of the deterministic models with an assumption of profit maximization.

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