Real Scores in Multiple Choice Tests for Large Item Cases

Surin Khanabsakdi * **and Thammarat Tuntisuntisom** **

1. Introduction

In multiple choice tests, students are tempted to choose the answers by guessing where they do not know the correct answer. Let T denote the total score, R the real score from knowledge - based answering, and X the random score from simply guessing. Thus, T = R+X. Thamchareon [6] first introduced the sampling distribution of T, R and X. Later Khanabsakdi et al. [3] studied the conditional expectation of R given T = t by using the test scores of all students who succeeded in passing the examination paper for further study in Chiang Mai University in the academic year 1995.

Generally, if the examination paper is standardized, the student scores will tend to normal by the law of large numbers (see [1]). In this paper, we study conditional expectation of R given T=t in multiple choice tests at large numbers of items. However, before proceeding to that point, we need a sampling distribution of the three random variables and some concepts.

2. Sampling distribution and concept

Each student answers a multiple choice test of n questions where each question has c choices. If the student answers any question by guessing, the probability of choosing the correct answer is 1/c.

2.1 Sampling Distribution of T. It is obvious that

$$P(T = t) = \frac{\text{number of students getting t points}}{\text{total number of students}}$$

2.2 Sampling Distribution of R. From Thamchareon [6], we have that

$$P(T=t) = \sum_{r=0}^{t} P(T=t, R=r)$$

$$= \sum_{r=0}^{t} P(R=r)P(T=t|R=r)$$

$$= \sum_{r=0}^{t} P(R=r)P(X=t-r|R=r)$$

$$P(R=r) = \frac{P(T = r) - \sum_{k=0}^{r-1} P(R = k)P(X = r - k|R = k)}{P(X = 0|R = r)}$$

---

* Associate professor, Research Unit of Probability Application, Department of Statistics,Faculty of Science, Chiang Mai University.

** Lecturer, Research Unit of Probability Application, Department of Statistics,Faculty of Science, Chiang Mai University.

2.3 Sampling Distribution of X. By using a binomial distribution mass function (see [4]), we have a conditional probability mass function of X, given R=r as follows :

$$P(X=x|R=r) = \binom{n-r}{x}\left(\frac{1}{c}\right)^{x}\left(1-\frac{1}{c}\right)^{n-r-x}, \quad x=0,1,2\ldots, n-r. \qquad (2.4)$$

2.4 Adjusted Probability. The probability of R=r by using the formula (2.3) may be negative which does not satisfy its property. It can be adjusted by the following procedure :

2.4.1 Add each probability value of R to the absolute least value.

2.4.2 Sum all the results of step 2.4.1.

2.4.3 Divide all the results of 2.4.1 by the summation of 2.4.2.

The results in 2.4.3 represent adjusted probability, denoted by P_a (R=r).

From (2.2), we also have to adjust the probability of T, denoted by P_a (T=t), as follows :

$$P_a(T=t) = \sum_{r=0}^{t} P_a(R=r)P(T=t|R=r).$$

2.5 Conditional Expectation and Variance of R, given T=t. By definition of conditional expectation and variance (see [2, 5]), we have a conditional expectation of R, given T=t, denoted by E(R|T=t), and a conditional variance of R, given T=t, denoted by V(R|T=t), as follows :

$$E(R|T=t) = \sum_{r=0}^{t} rP(R=r|T=t)$$

$$= \sum_{r=0}^{t} \frac{rP(T=t|R=r)P_a(R=r)}{P_a(T=t)} \qquad (2.6)$$

$$V(R|T=t) = E(R^2|T=t) - (E(R|T=t))^2. \qquad (2.7)$$

3. Method

Score and its frequency of multiple choice test are constructed under many value of the mean (μ) and standard deviation (σ) at large numbers of item (n) for c=2,3,4,5 ($n \geq 32$ for c=2, $n \geq 43$ for c=3, $n \geq 58$ for c=4, and $n \geq 72$ for c=5) and at small n also. The data are analysed via the procedure described previously using a computer program.

4. Results and discussion

In case of large n; at number of choice c and total score t of those among different μ and/or different σ, the conditional expected values of real score are equal and its standard errors are also equal. This paper only shows the conditional expected values of real score and its standard errors, given T=t, in Table 1. The conditional expected values are also shown in Figure1.

In case of small n; the results are different in both conditional expected values and its standard errors. The results of each value cannot be shown in this paper because its depend on value of μ and σ. However, its can be analysed via the procedure described previously.

The percentage of difference between total score and conditional expected real score is large when total score is small and it is small when total score is large. It is imply that the conditional expected real scores is good estimator when total score is large.

At large values of c, the conditional expected values of real score, given T=t, tends to t. It is not shown in this paper, however, it is obviously seen from its graph in Figure 1.

Table 1 Show E(R|T=t)at large mumber of n

T	c = 2		c = 3		c = 4		c = 5	
	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.02	0.14	0.03	0.17	0.04	0.19	0.05	0.21
2	0.04	0.20	0.06	0.24	0.08	0.28	0.10	0.31
3	0.06	0.25	0.09	0.31	0.12	0.36	0.16	0.40
4	0.08	0.29	0.13	0.37	0.17	0.43	0.22	0.49
5	0.11	0.34	0.16	0.42	0.22	0.50	0.29	0.57
6	0.13	0.37	0.20	0.48	0.28	0.57	0.37	0.66
7	0.16	0.41	0.24	0.53	0.34	0.64	0.45	0.75
8	0.18	0.45	0.29	0.59	0.41	0.72	0.55	0.84
9	0.21	0.49	0.34	0.64	0.48	0.79	0.65	0.94
10	0.24	0.53	0.39	0.70	0.57	0.87	0.77	1.05
11	0.27	0.57	0.44	0.76	0.65	0.96	0.91	1.17
12	0.30	0.60	0.50	0.82	0.75	1.05	1.06	1.30
13	0.33	0.64	0.53	0.89	0.86	1.15	1.24	1.44
14	0.37	0.69	0.63	0.96	0.99	1.26	1.44	1.59
15	0.40	0.73	0.71	1.03	1.12	1.38	1.67	1.75
16	0.44	0.77	0.79	1.11	1.28	1.50	1.93	1.93
17	0.48	0.81	0.88	1.19	1.45	1.64	2.24	2.12
18	0.52	0.86	0.98	1.28	1.64	1.79	2.59	2.33
19	0.57	0.91	1.08	1.38	1.87	1.95	3.00	2.55
20	0.61	0.96	1.20	1.48	2.12	2.12	3.47	2.79
21	0.66	1.01	1.33	1.59	2.41	2.31	4.01	3.03
22	0.72	1.06	1.47	1.71	2.74	2.52	4.63	3.28
23	0.77	1.12	1.63	1.84	3.11	2.74	5.33	3.52
24	0.83	1.18	1.80	1.97	3.54	2.97	6.11	3.75
25	0.90	1.24	2.00	2.12	4.04	3.21	6.98	3.96
26	0.97	1.31	2.22	2.29	4.60	3.46	7.93	4.14
27	1.04	1.38	2.47	2.46	5.23	3.72	8.96	4.30
28	1.12	1.45	2.74	2.65	5.95	3.98	10.04	4.42
29	1.20	1.53	3.06	2.85	6.75	4.22	11.18	4.50
30	1.30	1.62	3.41	3.07	7.64	4.45	12.36	4.56
31	1.39	1.71	3.82	3.31	8.61	4.66	13.56	4.58
32	1.50	1.80	4.27	3.56	9.66	4.84	14.78	4.59
33	1.62	1.90	4.79	3.82	10.78	4.99	16.02	4.58
34	1.74	2.01	5.37	4.09	11.95	5.09	17.26	4.56
35	1.88	2.13	6.03	4.37	13.18	5.17	18.50	4.53
36	2.03	2.25	6.77	4.65	14.44	5.21	19.75	4.50
37	2.19	2.39	7.60	4.93	15.73	5.23	21.00	4.47
38	2.37	2.53	8.52	5.20	17.03	5.23	22.25	4.44
39	2.57	2.69	9.53	5.45	18.35	5.22	23.50	4.40
40	2.79	2.85	10.62	5.67	19.68	5.19	24.75	4.37
41	3.03	3.04	11.80	5.86	21.00	5.15	26.00	4.33
42	3.30	3.23	13.05	6.02	22.34	5.12	27.25	4.29
43	3.60	3.44	14.37	6.14	23.67	5.07	28.50	4.26
44	3.94	3.67	15.74	6.22	25.00	5.03	29.75	4.22
45	4.31	3.92	17.15	6.27	26.33	4.99	31.00	4.18
46	4.74	4.18	18.59	6.29	27.67	4.94	32.25	4.15
47	5.21	4.46	20.05	6.28	29.00	4.90	33.50	4.11
48	5.75	4.76	21.53	6.25	30.33	4.85	34.75	4.07
49	6.36	5.08	23.01	6.22	31.67	4.81	36.00	4.03
50	7.04	5.42	24.51	6.17	33.00	4.76	37.25	3.99

Table 1 Show E(R|T=t)at large mumber of n (Continue)

T	c = 2		c = 3		c = 4		c = 5	
	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)	E(R|t)	σ(R|t)
51	7.81	5.76	26.00	6.12	34.33	4.71	38.50	3.95
52	8.67	6.12	27.50	6.06	35.67	4.67	39.75	3.91
53	9.64	6.48	29.00	6.00	37.00	4.62	41.00	3.87
54	10.72	6.83	30.50	5.94	38.33	4.57	42.25	3.83
55	11.91	7.17	32.00	5.87	39.67	4.52	43.50	3.79
56	13.23	7.49	33.50	5.81	41.00	4.47	44.75	3.75
57	14.65	7.78	35.00	5.74	42.33	4.42	46.00	3.71
58	16.19	8.02	36.50	5.68	43.67	4.37	47.25	3.67
59	17.83	8.22	38.00	5.61	45.00	4.32	48.50	3.62
60	19.56	8.36	39.50	5.55	46.33	4.27	49.75	3.58
61	21.36	8.46	41.00	5.48	47.67	4.22	51.00	3.54
62	23.23	8.50	42.50	5.41	49.00	4.16	52.25	3.49
63	25.14	8.50	44.00	5.34	50.33	4.11	53.50	3.45
64	27.08	8.47	45.50	5.27	51.67	4.06	54.75	3.40
65	29.04	8.40	47.00	5.20	53.00	4.00	56.00	3.53
66	31.02	8.32	48.50	5.12	54.33	3.94	57.25	3.31
67	33.01	8.22	50.00	5.05	55.67	3.89	58.50	3.27
68	35.01	8.11	51.50	4.97	57.00	3.83	59.75	3.21
69	37.00	7.99	53.00	4.90	58.33	3.77	61.00	3.16
70	39.00	7.87	54.50	4.82	59.67	3.71	62.25	3.11
71	41.00	7.74	56.00	4.74	61.00	3.65	63.50	3.06
72	43.00	7.62	57.50	4.66	62.33	3.59	64.75	3.01
73	45.00	7.48	59.00	4.58	63.67	3.53	66.00	2.96
74	47.00	7.35	60.50	4.50	65.00	3.46	67.25	2.90
75	49.00	7.21	62.00	4.42	66.33	3.40	68.50	2.85
76	51.00	7.07	63.50	4.33	67.67	3.33	69.75	2.80
77	53.00	6.93	65.00	4.24	69.00	3.27	71.00	2.74
78	55.00	6.78	66.50	4.15	70.33	3.20	72.25	2.68
79	57.00	6.63	68.00	4.06	71.67	3.13	73.50	2.62
80	59.00	6.48	69.50	3.97	73.00	3.06	74.75	2.56
81	61.00	6.32	71.00	3.87	74.33	2.98	76.00	2.50
82	63.00	6.16	72.50	3.77	75.67	2.91	77.25	2.44
83	65.00	6.00	74.00	3.67	77.00	2.83	78.50	2.37
84	67.00	5.83	75.50	3.57	78.33	2.75	79.75	2.30
85	69.00	5.66	77.00	3.46	79.67	2.67	81.00	2.24
86	71.00	5.48	78.50	3.35	81.00	2.58	82.25	2.17
87	73.00	5.29	80.00	3.24	82.33	2.49	83.50	2.09
88	75.00	5.10	81.50	3.12	83.67	2.40	84.75	2.02
89	77.00	4.90	83.00	3.00	85.00	2.31	86.00	1.94
90	79.00	4.69	84.50	2.87	86.33	2.21	87.25	1.85
91	81.00	4.47	86.00	2.74	87.67	2.11	88.50	1.77
92	83.00	4.24	87.50	2.60	89.00	2.00	89.75	1.68
93	85.00	4.00	89.00	2.45	90.33	1.89	91.00	1.58
94	87.00	3.74	90.50	2.29	91.67	1.76	92.25	1.48
95	89.00	3.46	92.00	2.12	93.00	1.63	93.50	1.37
96	91.00	3.16	93.50	1.94	94.33	1.49	94.75	1.25
97	93.00	2.83	95.00	1.73	95.67	1.33	96.00	1.12
98	95.00	2.45	96.50	1.50	97.00	1.15	97.25	0.97
99	97.00	2.00	98.00	1.22	98.33	0.94	98.50	0.79
100	99.00	1.41	99.50	0.87	99.67	0.67	99.75	0.56

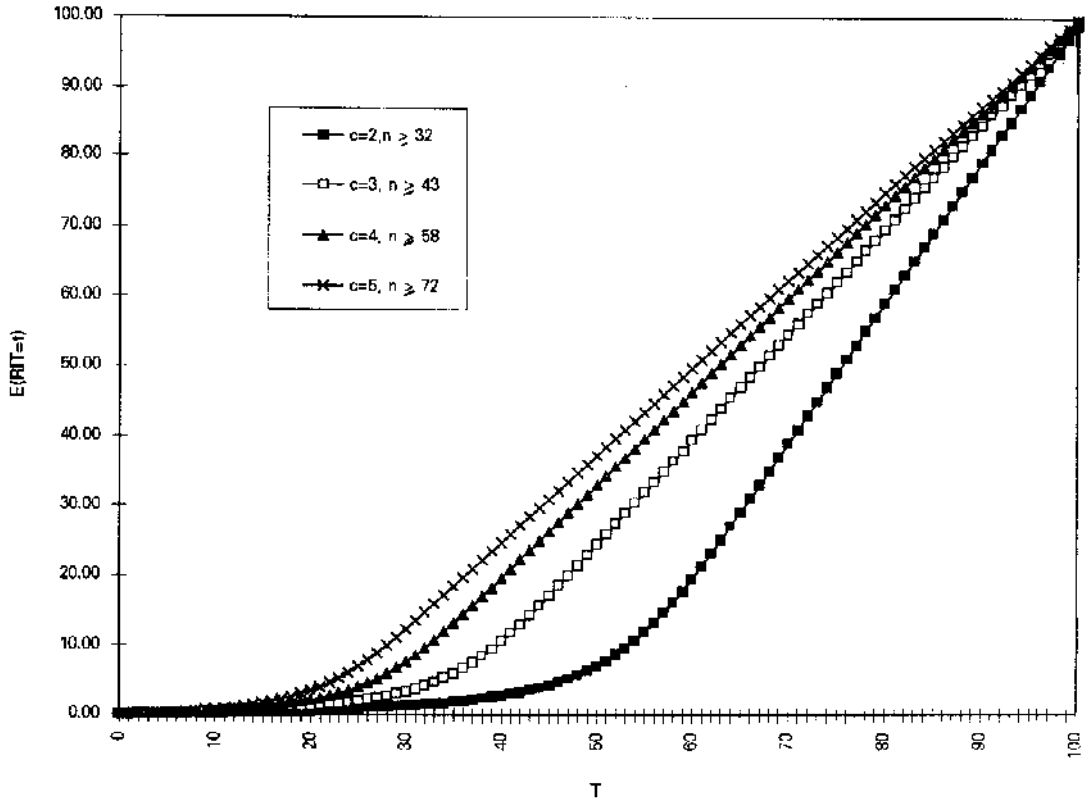

Fig.1 Graph shows real score R in term of expectancy at total score T=t

5. Conclusions

When students are tested using a multiple choice test in which the number of items is large, answers are chosen based on knowledge where the student thinks that he/she knows the answer or by simply guessing where he/she does not. One point is awarded for each correct answer and none for each incorrect answer. For a total score of T=t, a student's real score, corresponding to his/her real knowledge, is analysed in terms of expectancy as shown in Table 1 and Figure1. For example, if a student gets 80 points in total score, for 2 choices in each item, the expected real score from knowledge-base in 59.00 points or the guessing score is 21 points with standard error is 6.48 points.

6. Acknowledgements

The authors would like to thank Dr. Robert Molloy, Faculty of Science, Chiang Mai University, for his reading of the manuscript and valuable suggestions for its improvement and Associate Professor Sanan Supasai and Associate Professor Wiyada Tanvattanagul, Department of Statistics, Faculty of Science, Chiang Mai University, for their comments on the statistical approach.

References

[1] Feller, W. *An Introduction to Probability Theory and its Applications,* 3rd Edn., New York : John Wiley and Sons, 1968.

[2] Freund, J.E. and Walpole, R.E. *Mathematical Statistics,* 4th Edn., New Jersey : Prentice-Hall, 1987.

[3] Khanabsakdi, S., Supasai, S., Saenchan, L. and Tanvatanagul, W. *Real Score on Multiple Choice Test.* Al-Azhar Bullentin of Science: Proc. 2nd International Scientific Conference, Faculty of Science, Al-Azhar University, Cairo, Egypt, 17-20 March 1997, 305-316.

[4] Olkin, I., Gleser, L.J. and Derman, C. *Probability Models and Applications,* 2nd Edn., New York : Macmillan College Publishing Company, 1994.

[5] Pfeiffer, P.E. *Probability for Applications*, New York : Springer-Verlag, 1990.

[6] Thamchareon, A. *The Distribution of the Real Score of the Multiple Choice Test with Guessing*, Proc. 9th Applied Statistics Conference, School of Applied Statistics, National Institute of Development Administration, Bangkok, 3-4 August 1992 (supplementary issue).